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Abstract

This paper shows that when children's ability is heterogeneous, a parent's choices about educational expenditures and fertility follow a pooling equilibrium or a separating equilibrium. Which of the two equilibria will prevail depends on the probability of getting a high-ability child in human capital accumulation. Adopting the model of Acemoglu's (1999), this paper presents that the outcome of the pooling choice in the pooling regime and the outcome of the separating choice in the separating regime make the growth rate of human capital higher than otherwise. In addition, as the probability of a high-ability child increases, the growth rate of human capital in the separating equilibrium exceeds that in the pooling equilibrium.

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I. Introduction

The basic concept of human capital as an input in production was, for the first time, introduced by Adam Smith. He defined human capital as "an acquired and useful abilities of all the inhabitants or members of the society." Since then, much work in the literature such as Mincer (1958), Schultz (1961), Becker (1962, 1964, 1981, 1993), and Ben-Porath (1967) has been done, and they have contributed greatly to the developments and its applications of human capital theory.

Mincer (1958) shows theoretically that higher wage requires more training. Considering that the distribution of income is right-skewed, while the distribution of individual abilities is Gaussian normal distribution, he concludes that inter-occupational differentials in income would depend on differences in training, and that intra-occupational differentials are related to experiences.

Stressing human capital as a form of capital, Schultz (1961) states that "direct expenditures on education, health, and internal migration, and attending school and on-the-job training" are investments in human capital. He points out that the growth rates in the U.S. income are higher than those of land, work hours of workers, and reproducible capital stock, and that the occurrence of this discrepancy between outputs and inputs is due to omission of improvements in human capital. That is, this discrepancy implies that investments in human capital are an important factor in explaining economic growth. Actually, the economic miracles in Hong Kong, Korea, Taiwan, and other Asian countries (the so-called Asian tigers), with few natural resources, show the importance of labor quality as an engine of economic growth.

In addition to the positive impacts of human capital on economic growth, Becker (1981) emphasizes a parental role in developing the human capital of children. In his book, *A Treatise on the Family*, he argues that most parents decide how to invest in their children's human capital, and the decisions may be different, depending upon the children's innate ability. Specifically, children of more cognitive ability would have a higher rate of returns to education than others, and thus parents are likely to invest more to accumulate their children's human capital.

In recent years, many economists in the literature have contributed to human capital theory, but most of them assume that children's ability is homogeneous. Differently with the previous studies, we explore parental choices on educational expenditures and fertility with heterogeneous ability of their children, as in Becker (1981). On the other hand, we adopt Acemoglu's (1999) notion that there exist two equilibria—pooling equilibrium and separating equilibrium—when a firm faces randomly workers, with skills heterogeneous, in the labor market. Specifically, in his model, a pooling equilibrium occurs when the firm produces with a recruited worker and chooses the same level of physical capital for all workers, regardless of whether the worker is skilled or unskilled. In the separating equilibrium, however, the firm chooses larger capacity (capital) for skilled workers. If the firm faces an unskilled worker, then it turns him down and shuts down. These different choices are determined by the conditions of labor market (e.g., the fraction of skilled workers in the labor market and the degree of productivity differentials between skilled and unskilled workers).

With this notion, we attempt to show how the heterogeneity of children's ability can make their parents choose different types of investment in education and fertility. To do this, we assume that parents observe that, in an economy, there is a certain fraction of high-ability children and the degree of productivity differentials in human capital accumulation at time t. In this paper, we consider the fraction of high-ability children as a level of social human capital, exogenously given in an economy. Using this knowledge, they choose a type of investment in educating their children.

Given these conditions, once they choose a pooling type, they do not care about their children's ability to accumulate human capital. On the other hand, for a separating type, finding out the ability of their children, they will make discriminatory decisions on the children's human capital investment, depending upon the children's innate ability. In practice, parents can continue to observe their children's academic ability at home or kindergarten. Then they can use information on their children's ability when deciding how to invest for their children entering a school or going up to an advanced school. Once a type of investment in education is chosen, then a parent allocates her time and resources to consumption and education.

When a type of investment chosen by parents depends on the fraction of high-ability children and their productivity in human capital accumulation, the fraction of high ability can be interpreted various ways: for instance, we can simply regard it as a fraction of innate high-ability children born at time t in an economy. Also, it might be possible to interpret it as a function of social human capital (with positive relationship) because higher social human capital is more likely that children have high productivity in accumulating their human capital. The latter interpretation can be better and more useful to compare types of investment in education across countries.

Compared to the separating choice, the pooling choice can equalize educational opportunity. A parent who educates her child without considering her child's ability likely intends to provide public or compulsory education for her child. In contrast, if a child is judged to be smart, he will get private education in the discriminatory separating choice. Otherwise, he will get public education or no formal education.

If the fraction of high-ability children is relatively low given the productivity differential, a parent's equilibrium choice on education can be pooling. In practice, a parent can have no incentive to judge her child's ability with the likelihood of high ability and/or the productivity differential low. Furthermore, interpreting the fraction of high-ability children as a function of social human capital, the resulting pooling choice is consistent with the Glomm and Ravikumar's (1992) prediction that the societies where the majority of people have incomes below average, i.e., a low level of social human capital, will choose public education.

This paper is organized as follows: Section II presents a simple model of a parent's choice on educational expenditures to accumulate her child's human capital, along with the likelihood of high ability. A parental choice on fertility is added to the simple model in Section III. Section IV concludes.

II. Simple Model

1. The Setup

In this section, we consider an overlapping generations model in which each person lives only for two periods: childhood and adulthood. For clarity, the timing of events in the model is as follows: in childhood, she is educated, and then she works and educates her child in adulthood. To identify a parental choice on types of educating her child in the adulthood, there is a need to set up two sub-stages: at the first sub-stage, she observes her child's ability, and then immediately, at the second sub-stage, decides which type of education she will invest in. Specifically, as said earlier, her child receives ordinary rudimentary education at the first sub-stage, for instance, at home or kindergarten, irrespective of the parent's decision on education. Then at the second sub-stage, she decides on the type of education, though without certainty, based upon information on her child's ability. Furthermore, suppose that at the first sub-stage, a parent's child accumulates a minimal level of human capital, leading only to low incomes in the adulthood. To simplify the algebra, we assume that the minimal level of human capital is normalized to zero, with no costs at the first sub-stage. Together with all these assumptions, we focus on the optimization problem at the second sub-stage—we call it at time $t+1$—with educating children who have heterogeneous abilities.

For simplicity, we define a parent's utility function as a linear form:[1]

$$c_t + \beta h_{t+1}, \tag{1}$$

where c_t is the parental consumption at time t, h_{t+1}, her child's human capital, and β, a weight on the human capital. Assume that the parent cares only about her own consumption and her child's human capital. She does not care about the human capital or utility of the offspring of her child. The objective function in Equation (1) implies that a parent has only one child. The budget constraint given to the parent can be written as

[1] Intuitively, it can make more sense that a parent' utility for her child comes from the child's happiness or welfare, rather than human capital. However, we simply assume that her child's human capital is a proxy for happiness.

$$c_t + e_{t+1} \leq w_t h_t (1-\theta). \tag{2}$$

The parent's capacity for work is normalized to unity. Assume that the labor market is competitive, and then w_t is an equilibrium wage per unit of human capital in the labor market. Thus, the capacity earnings are $w_t h_t$ for a person who has human capital h_t, at time t. She invests e_{t+1} from her earnings in educating her child at time t. On the other hand, the time cost required to rear a child is assumed to be a fraction of time, $\theta \in (0,1)$, but it does not contribute to the child's human capital. More specifically, a parent allocates her time to doing work and rearing a child, and then does her effective earnings from work to her own consumption and the education of her child.

Relying heavily on Acemoglu (1999) for the theoretical analysis, the law of motion for human capital accumulation can be defined by

$$h_{t+1} = \lambda\, d^H h_t^{\delta} e_{t+1}^{\alpha} \kappa^{\gamma} + (1-\lambda) d^L h_t^{\delta} e_{t+1}^{\alpha}, \tag{3}$$

where $\lambda \in (0,1)$ is a probability of having a high-ability child, which can be measured by a level of social human capital in an economy. For simplicity, we normalize to unity the productivity of a low-ability child in human capital accumulation, while the productivity of a high-ability child κ is greater than unity. Also, assume that $\alpha \in (0,1)$, and δ and γ are both positive. Here all these parameters $\alpha, \delta,$ and γ are the elasticity of the child's human capital with respect to educational expenditures, parental human capital, and its high-ability, respectively. Specifically, $h_t^{\delta} e_{t+1}^{\alpha} \kappa^{\gamma}$ in Equation (3) is the human capital production function for a high-ability child, while $h_t^{\delta} e_{t+1}^{\alpha}$, for a low-ability child. The variable d^j is a parent's decision variable on whether to invest or not in educating her child for each state, high ability, H, and low-ability, L. We define the space of the decision variables by a continuum of choice between zero and unity inclusive, i.e., $d^j \in [0,1]$ for $j = H, L$.

We highlight that a parent's choice in educational investment occurs after defining her child's ability, aforementioned two sub-stages in the adulthood. Then we assume that a parent simply knows ex ante the likelihood, λ, and the productivity, κ, given exogenously. In practice, she might estimate λ by measuring a level of social human capital in her country. These two variables λ and $\kappa > 1$ imply that there exists a positive externality of social human capital in accumulating her child's human capital. As a result, considering the two factors λ and κ, she chooses a type of investment in education after observing her child's ability. Together with λ in Equation (3), we can see that the law of motion is an expected production function for a child's human capital. Also, a parent's human capital, her child's ability, and the educational expenditures are employed as inputs to produce the child's human capital.

2. Two Equilibria and Characteristics

Using Equations (1), (2), and (3), the expected value for a parent's decision problem can be expressed as

$$V^{*} = \max_{\{e_{t+1}, d^{H}, d^{L}\}}[w_t h_t (1-\theta) - e_{t+1}(\lambda\, d^{H} + (1-\lambda) d^{L}) + \beta\{\lambda\, d^{H} h_t^{\delta} e_{t+1}^{\alpha} \kappa^{\gamma} + (1-\lambda) d^{L} h_t^{\delta}\, e_{t+1}^{\alpha}\}]. \tag{4}$$

Since the parent's decision on educational investment depends on λ and κ, the expected expenditures on education allocated from effective earnings can be rewritten as a linear combination of the likelihood of ability and decision variables for each state. As said earlier, if a parent's decision on investment in education for her child is independent of whether her child's ability is high or low, call it "pooling." Otherwise, call it "separating." That is, if $d^{H} = d^{L}$, then it is called pooling, and if $d^{H} \neq d^{L}$, then separating.

Differentiating Equation (4) with respect to e_{t+1}, we obtain the following first-order condition:

$$\alpha\beta h_t^{\delta} e_{t+1}^{\alpha-1}\{\lambda d^{H}\kappa^{\gamma} + (1-\lambda) d^{L}\} = \lambda d^{H} + (1-\lambda) d^{L}. \tag{5}$$

The left-hand side in Equation (5) shows the marginal benefits of investing one unit of resource in education, while the right-hand side, the marginal costs. Since Equation (4) is concave in expenditures on education, e_{t+1}, due to $\alpha \in (0,1)$, the necessary condition in Equation (5) satisfies the sufficient second-order conditions. Solving Equation (5) for e_{t+1}, we can obtain the parent's optimal educational expenditures as follows:

$$e_{t+1}^{*} = \left[\frac{\alpha\beta h_t^{\delta}\{\lambda d^{H}\kappa^{\gamma} + (1-\lambda) d^{L}\}}{\lambda d^{H} + (1-\lambda) d^{L}}\right]^{\frac{1}{1-\alpha}}. \tag{6}$$

Additionally, plugging Equation (6) into Equation (4), we can, in turn, get the following optimization problem:

$$V^{*} = \max_{\{d^{H}, d^{L}\}}\left[w_t h_t (1-\theta) + \frac{1-\alpha}{\alpha}\left[\frac{\alpha\beta h_t^{\delta}\{\lambda d^{H}\kappa^{\gamma} + (1-\lambda) d^{L}\}}{\{\lambda d^{H} + (1-\lambda) d^{L}\}^{\alpha}}\right]^{\frac{1}{1-\alpha}}\right]. \tag{7}$$

Next, we compare the maximal values of the parent's problem in Equation (7), depending upon the decision variable, $d^{j} \in [0,1]$ for $j = H, L$, to find out the equilibria.

Proposition 1 *If* $\kappa > [(1-\lambda)/(\lambda^{1-\alpha}-\lambda)]^{1/\gamma}$, *then the equilibrium is unique and separating with* $(d^H, d^L) = (1{,}0)$, *where the optimal expenditures on education are* $e_{t+1}^{s^*} = \left[\alpha\beta\, h_t^{\delta}\kappa^{\gamma}\right]^{1/(1-\alpha)}$. *Otherwise, then it is unique and pooling with* $(d^H, d^L) = (1{,}1)$, *where they are* $e_{t+1}^{p^*} = \left[\alpha\beta\, h_t^{\delta}\{\lambda\kappa^{\gamma} + (1-\lambda)\}\right]^{1/(1-\alpha)}$.[2]

Proof. See Appendix.

Figure 1 shows the critical condition, $\kappa = [(1-\lambda)/(\lambda^{1-\alpha}-\lambda)]^{1/\gamma}$, in Proposition 1. The critical condition shows that the high ability, $\kappa > 1$, is downward sloping in the probability λ, as in Acemoglu (1999). When given $\kappa > 1$, the probability of high-ability child, λ, measured by a level of social human capital, increases and, in turn, exceeds the critical point, the parent's decision on educational investment switches from the pooling to the separating equilibrium. In addition, for $\lambda \in (0{,}1)$, expected expenditures on education in the separating equilibrium, $\lambda e_{t+1}^{s^*}$, is greater than, $e_{t+1}^{p^*}$, in the pooling equilibrium if an economy is in the separating equilibrium, and vice versa. In other words, the expected education expenditures are maximal in the choice yielding the equilibrium in each regime.

[Figure 1 about here]

The separating equilibrium can imply higher inequality in education, leading to higher inequality in income, relative to the pooling equilibrium. With respect to the two equilibria, Barro (1999) argues that higher inequality in income slows down the growth in poor countries, while it accelerates the growth in rich countries. This implies that each of these two equilibria is optimal under its own regime in terms of economic growth.

In Figure 2, the red solid line is a standardized expected educational expenditure of a parent's pooling choice of $(d^H, d^L) = (1{,}1)$ with λ, while the blue dot-dashed line, of the parent's separating choice of $(d^H, d^L) = (1{,}0)$. The point where the two lines intersect is the critical probability, λ^*, which switches regimes from the pooling to the separating equilibrium. In Figure 2, together with the calibrations of $\alpha\beta\, h_t^{\delta} = 1$, $\alpha = 0.5$, $\gamma = 0.4$, and $\kappa = 10$, the critical probability, λ^*, appears to be about 0.43.

[2] This proposition is similar to Acemoglu (1999).

[Figure 2 about here]

As shown in Figure 2, the expected educational expenditures increase with the probability, λ, for both cases. They are higher in each regime than otherwise, although this result is different from that in the model containing fertility in the next section. If the probability, λ, becomes higher, then an increase in contribution to the value function in Equation (4) from the expected expenditures on education for the two equilibria is greater than a decrease in contribution from the reduced consumption because of $\kappa > 1$. Furthermore, we can see that with the higher probability, the expected expenditures on education in the separating equilibrium are higher than in the pooling equilibrium, suggesting that parents in an economy with a high level of social human capital are more willing to spend resources on education than with a low level of social human capital.

Meanwhile, we can also see in Proposition 1 that the expected expenditures on education increase with a level of high ability and its intensity κ^{γ} for both the cases. An increase in κ would make the production of human capital more productive, leading to an increase in parental expenditures on education. With Proposition 1, the higher ability is or productivity differentials are, the greater the differentials in the expected educational expenditures between the two equilibria as in Equation (8). This implies that the differentials in human capital accumulation become increasingly high, leading to greater inequalities between the two equilibria.

$$\frac{\partial}{\partial \kappa^{\gamma}}\left(\frac{\lambda e_{t+1}^{*s}}{e_{t+1}^{*p}}\right) = \frac{1}{1-\alpha}\left[\frac{\lambda^{1-\alpha}\kappa^{\gamma}}{\lambda\kappa^{\gamma}+(1-\lambda)}\right]^{\frac{\alpha}{1-\alpha}} \frac{1-\lambda}{(\lambda\kappa^{\gamma}+(1-\lambda))^{2}} > 0 \tag{8}$$

More clearly, a higher dispersion of children's ability leads to higher educational expenditures in the separating equilibrium than in the pooling equilibrium.

In Proposition 1, the expected educational expenditures in both equilibria increase with the elasticity of human capital production with respect to investments in education, α, the weight on utility from the child's human capital, β, and a parent's human capital and its intensity, h_t^{δ}. Besides, the rearing costs, θ, in Equation (4) play no role in determining the expenditures. Fertility is normalized to one in Equation (4), even though the given parent's time cost, θ, has effects on fertility because, in turn, fertility affects educational expenditures.

Though shown in the next section, we will point out here that an increase in rearing costs makes rearing children more expensive to parents of high human capital, relative to those of low human capital. Thus increased rearing costs would have parents of high human capital get more incentives to reduce the number of children. This result is well shown in Becker, Murphy, and Tamura (1990).

3. Characteristics in Dynamics of Two Equilibria

Substituting the optimal educational expenditures into Equation (3), we examine the dynamics of human capital for each of equilibria. Under pooling and separating regimes, the laws of motion for human capital are expressed as

$$h_{t+1}^{p^*} = (\alpha\beta)^{\frac{\alpha}{1-\alpha}} h_t^{\frac{\delta}{1-\alpha}} [\lambda\kappa^{\gamma} + (1-\lambda)]^{\frac{1}{1-\alpha}}, \tag{9}$$

$$h_{t+1}^{s^*} = (\alpha\beta)^{\frac{\alpha}{1-\alpha}} h_t^{\frac{\delta}{1-\alpha}} [\lambda^{1-\alpha}\kappa^{\gamma}]^{\frac{1}{1-\alpha}}. \tag{10}$$

[Figure 2 about here]

Provided that $\delta/(1-\alpha) < 1$, the first-order difference equations in terms of human capital have the unique and stable steady-state equilibrium, respectively. If $\delta/(1-\alpha) = 1$ and neither $(\alpha\beta)^{\frac{\alpha}{1-\alpha}}[\lambda\kappa^{\gamma} + (1-\lambda)]^{\frac{1}{1-\alpha}}$ nor $(\alpha\beta)^{\frac{\alpha}{1-\alpha}}[\lambda^{1-\alpha}\kappa^{\gamma}]^{\frac{1}{1-\alpha}}$ is unitary, we have no steady-state point. On the other hand, if $\frac{\delta}{1-\alpha} > 1,$ we have steady-state points for both the regimes, but they are unstable, causing multiple long-run equilibria in terms of human capital accumulation.

Together with $\alpha = 0.5,$ $\beta = 0.5,$ $\gamma = 0.4,$ $\kappa = 10,$ $\lambda_1 = 0.3,$ and $\lambda_2 = 0.7,$ Panel (a) in Figure 3 shows the case of $\delta/(1-\alpha) < 1,$ Panel (b), $\delta/(1-\alpha) = 1,$ and Panel (c), $\delta/(1-\alpha) > 1.$ Interestingly, along with these calibrations of parameters, Panel (b) shows that the long-run equilibrium of human capital is zero under the pooling choice due to low probability, while it is infinity under the separating choice due to high probability. This implies that the probability can result in different long-run equilibria of human capital. Figure 3 shows that the rate of growth of human capital in the separating equilibrium is higher than in the pooling equilibrium because of the higher probability of high ability in the separating regime.

[Figure 3 about here]

Equations (9) and (10) show that when given $\kappa > 1$, the probability of high ability is sufficiently low, it is reasonable for a parent to choose a pooling decision, leading to higher growth rate of human capital than the separating decision. However, as the probability grows sufficiently high, a parent's discriminatory education decision across children's ability is efficient, leading to switching to the separating equilibrium and thus to higher economic growth. That is, it is inefficient for a parent to invest the same amount of educational expenditures, irrespective of her child's ability when the probability is sufficiently high—a high level of social human capital.

III. The Model with Fertility

1. The Setup

In this section, we add fertility to the simple model in Section II. A parent's utility function can be defined as

$$c_t + \beta n_t^{1-\epsilon} h_{t+1}^{\epsilon}, \tag{11}$$

where n_t is a parental fertility at time t with $0 < \epsilon < 1$. The second term of Equation (11), which has a Cobb-Douglas form, shows that quantity and quality of children are complementary to a parent's utility. The budget constraint below is similar to that in the simple model setup except for fertility:

$$c_t + n_t e_{t+1} \leq w_t h_t (1 - \theta n_t), \tag{12}$$

where the total time costs for a parent's rearing n_t children at time t are θn_t with the costs of rearing a child, θ. Since we assume that children's ability within a household is homogeneous, it is efficient that the parent, a head in the given household, treats her children equally. And thus, we do not consider a possibility that she differentiates her children. She makes investment decisions on education for her children, depending upon λ and κ. We employ the same law of motion for human capital as in Section II, i.e., Equation (3).

2. Two Equilibria and Characteristics

Using Equations (3), (11) and (12), the household's decision problem can be expressed as follows:

$$\begin{aligned} V^* = \max_{\{e_{t+1}, n_t, d^H, d^L\}} [w_t h_t (1 - \theta n_t) - n_t e_{t+1} (\lambda\, d^H + (1-\lambda) d^L)\} \\ + \beta n_t^{1-\epsilon} \{\lambda d^H h_t^{\delta} e_{t+1}^{\alpha} \kappa^{\gamma} + (1-\lambda) d^L h_t^{\delta} e_{t+1}^{\alpha}\}^{\epsilon}], \end{aligned} \tag{13}$$

Differentiating Equation (13) with respect to e_{t+1} and n_t, we can obtain the following necessary conditions on educational expenditures the parental fertility:

$$\beta \alpha \epsilon\, n_t^{1-\epsilon} h_t^{\delta\epsilon} e_{t+1}^{\alpha\epsilon - 1} [\lambda d^H \kappa^{\gamma} + (1-\lambda) d^L]^{\epsilon} = n_t (\lambda\, d^H + (1-\lambda) d^L) \tag{14}$$

$$\beta (1-\epsilon) n_t^{-\epsilon} h_t^{\delta\epsilon} e_{t+1}^{\alpha\epsilon} [\lambda d^H \kappa^{\gamma} + (1-\lambda) d^L]^{\epsilon} = \theta w_t h_t + e_{t+1} (\lambda\, d^H + (1-\lambda) d^L) \tag{15}$$

The left-hand side in Equation (14) shows the marginal benefits from increasing one unit of educational expenditures, while the right-hand side, the marginal costs decreasing the expected value.

Likewise, the left-hand side in Equation (15) is the marginal benefits of having a child, while the right-hand side, the marginal costs, the costs of rearing and educating a child on the margin. Since $0 < \alpha < 1$ and $0 < \epsilon < 1$, the parent's problem in Equation (13) is jointly concave in educational expenditures, e_{t+1}, and the number of children, n_t, which implies that the sufficient conditions hold.

Solving for e_{t+1} and n_t in Equations (14) and (15), we can get the following optima for these variables:

$$e_{t+1}^{*} = \frac{\alpha\epsilon\theta w_t h_t}{1-\epsilon-\alpha\epsilon} \times \frac{1}{\lambda\, d^H + (1-\lambda) d^L}, \tag{16}$$

$$n_t^{*} = \left[\left(\frac{1-\epsilon-\alpha\epsilon}{\alpha\epsilon\theta w_t}\right)^{1-\alpha\epsilon} \alpha\beta\epsilon h_t^{\epsilon(\alpha+\delta)-1}\right]^{\frac{1}{\epsilon}} \times \frac{\lambda d^H \kappa^{\gamma} + (1-\lambda) d^L}{\{\lambda d^H + (1-\lambda) d^L\}^{\alpha}}. \tag{17}$$

Using Equations (13) and Equation (15), we can write the following value function:

$$V^{*} = w_t h_t \left[1 + n_t^{*}\left(\frac{\epsilon\theta}{1-\epsilon-\alpha\epsilon}\right)\right] \tag{18}$$

Together with the condition $0 < \epsilon < 1/(1+\alpha)$, we can compare the maximal values of Equation (18), depending upon the decision variable, $d^j \in [0,1]$ for $j = H, L$. Then we can obtain the following proposition.

Proposition 2 *If* $\kappa > \left[\frac{1-\lambda}{\lambda^{1-\alpha}-\lambda}\right]^{\frac{1}{\gamma}}$, *then the equilibrium is unique and separating with* $(d^H, d^L) = (1,0)$, *where the optimal expenditures on education and parental fertility are* $e_{t+1}^{s^*} = \frac{\alpha\epsilon\theta w_t h_t}{\lambda(1-\epsilon-\alpha\epsilon)}$ *and* $n_t^{s^*} = \left[\left(\frac{1-\epsilon-\alpha\epsilon}{\alpha\epsilon\theta w_t}\right)^{1-\alpha\epsilon} \alpha\beta\epsilon h_t^{\epsilon(\alpha+\delta)-1}\right]^{\frac{1}{\epsilon}} \lambda^{1-\alpha}\kappa^{\gamma}$. *Otherwise, then it is unique and pooling with* $(d^H, d^L) = (1,1)$, *where they are* $e_t^{p^*} = \frac{\alpha\epsilon\theta w_t h_t}{1-\epsilon-\alpha\epsilon}$ *and* $n_t^{p^*} = \left[\left(\frac{1-\epsilon-\alpha\epsilon}{\alpha\epsilon\theta w_t}\right)^{1-\alpha\epsilon} \alpha\beta\epsilon h_t^{\epsilon(\alpha+\delta)-1}\right]^{\frac{1}{\epsilon}} [\lambda\kappa^{\gamma} + (1-\lambda)]$.

Proof. To search for the equilibria, it is sufficient to simply compare $\frac{\lambda d^H \kappa^{\gamma} + (1-\lambda) d^L}{\{\lambda d^H + (1-\lambda) d^L\}^{\alpha}}$ in n_t^{*} for $d^j \in [0,1]$ for $j = H, L$. These comparisons are exactly the same as those in Appendix. Q.E.D.

As in Proposition 1, if given $\kappa > 1$, the probability that a high-ability child is born rises and then exceeds the critical point due to the downward-sloping critical condition, a parent's educational decision for her child switches from the pooling to the separating equilibrium. In Proposition 2, we can see that the parent's expected expenditures on education per child, $\lambda e_{t+1}^{s^*}$ and $e_{t+1}^{p^*}$, are identical in the pooling and separating equilibria, regardless of the probability λ. Instead, we get the result that optimal fertility is higher in its own regime.

More specifically, when the probability of high ability is sufficiently low, leading to a pooling equilibrium, the pooling choice for fertility is higher than the separating one. In contrast, when the probability is sufficiently high, leading to a separating equilibrium, the separating choice for fertility is higher than the pooling choice. However, a parent's total expected expenditures on education, $\lambda n_t^{s^*} e_{t+1}^{s^*}$ are higher in the separating equilibrium than, $n_t^{p^*} e_{t+1}^{p^*}$, in the pooling equilibrium.

Intuitively, when the probability that a born child has high ability is low, a parent's separating choice implies that fertility is low relative to her pooling choice, because if it is more likely that her children's ability is low, she is forced to take a risk of not educating them under the separating choice in the pooling regime. As a result, in an economy with this low probability, it is more efficient that a parent makes a pooling choice, not considering children's ability. Note that the pooling choice of fertility is modestly increasing in the probability λ.

In contrast, if the probability is sufficiently high, the marginal benefits of getting a child are increasing rapidly relative to the pooling choice, leading to a rapid increase in fertility because $\kappa > 1$. It implies that the separating choice has a higher positive externality from having children, compared to the pooling. Thus, in this case the separating choice is the equilibrium. It is worth noting that in either of the two equilibria, fertility is increasing in probability. As the probability goes up, a parent has an incentive to have more children since the risk of having low-ability children is decreasing and she can have positive externality from children with high ability in terms of her value function.

Consider educational and fertility choices in each equilibrium from Proposition 2 with respect to some exogenous variables. From the results in Proposition 2, first we have a constraint, $0 < \epsilon < 1/(1+\alpha)$, to avoid getting zero or negative educational expenditures and fertility. Also, the optimal educational expenditures are increasing in ϵ and θ. Since ϵ is an exponential weight in human capital, it appears to be natural to be positively correlated with the expenditures. More importantly, the rearing costs, θ, are also positively correlated to the expenditures, but negatively correlated with fertility. Increased rearing costs make children more expensive, and thus parents have incentives to spend more on education for more expensive children.

Meanwhile, if $\delta < 1$, the fertility choices in each regime are negatively correlated to parental human capital. Consistently with previous literature, this result is because the time costs of a parent with high human capital are large relative to low parental human capital from rearing children. On the other hand, it is straightforward to see that educational expenditures for both the regimes are positively correlated to the parental human capital.

Provided that the probability of high ability is due to social human capital and $0 < \epsilon < 1/(1+\alpha)$, the effects of social human capital and parental human capital on fertility are conflicting. Moreover, assuming that parental human capital is homogeneous at time t, whether or not fertility will increase as an economy grows is ambiguous. But empirically, fertility appears to decrease when an economy takes off from Malthusian stagnation to sustained economic growth (Galor, 2004).

3. Characteristics in Dynamics of Two Equilibria

Plugging the optimal educational expenditures into Equation (3) as in Section II, we can look through dynamics of human capital for each regime. Under pooling and separating regimes, the laws of motion for human capital are as follows:

$$h_{t+1}^{p^*} = \left(\frac{\alpha\epsilon\theta w_t}{1-\epsilon-\alpha\epsilon}\right)^{\alpha} h_t^{\alpha+\delta}[\lambda\kappa^{\gamma} + (1-\lambda)], \tag{19}$$

$$h_{t+1}^{s*} = \left(\frac{\alpha\epsilon\theta w_t}{1-\epsilon-\alpha\epsilon}\right)^{\alpha} h_t^{\alpha+\delta}\lambda^{1-\alpha}\kappa^{\gamma}. \tag{20}$$

Similar to the previous section, if $\alpha + \delta < 1$, then the first-order difference equations for human capital show that they both have a unique and stable steady-state equilibrium. If $\alpha + \delta = 1$ and neither $\left(\frac{\alpha\epsilon\theta w_t}{1-\epsilon-\alpha\epsilon}\right)^{\alpha}[\lambda\kappa^{\gamma} + (1-\lambda)]$ nor $\left(\frac{\alpha\epsilon\theta w_t}{1-\epsilon-\alpha\epsilon}\right)^{\alpha}\lambda^{1-\alpha}\kappa^{\gamma}$ is one, then there is no steady-state point. On the other hand, although if $\alpha + \delta > 1$, then we have steady-state points for both the regimes, we can see that they are unstable, leading to multiple long-run equilibria in terms of human capital accumulation.

Letting $A \equiv \left(\frac{\alpha\epsilon\theta w_t}{1-\epsilon-\alpha\epsilon}\right)^{\alpha}$, we see that the growth rate of human capital is higher in each own regime, regardless of the size of $\alpha + \delta$. However, note that due to higher probability, the growth rate of human capital in the separating equilibrium is higher than that in the pooling equilibrium as in Section II. That is, as the probability of high ability goes up, parents switch their choices from the pooling to the separating one, and then the growth rate of output in the separating equilibrium is higher than in the pooling equilibrium.

IV. Concluding Remarks

In this paper, we deal with parental choices about expected educational expenditures and fertility when children's innate ability is heterogeneous. Together with the heterogeneity in ability and the probability of high ability, a parent's choices create pooling and separating equilibria. When the probability of high ability is sufficiently low, the parent's pooling choice is optimal. In contrast, when it is sufficiently high, the separating choice is optimal. This implies that as heterogeneity and probability increase, the parent switches her choice from pooling to separating.

Assuming that a parent has only one child, we can see that expected educational expenditures for the pooling choice are larger than for the separating choice when an economy is in the pooling equilibrium. Likewise, when in the separating equilibrium, expenditures for the separating choice are larger. However, due to the higher probability, the growth rate of human capital in the separating equilibrium is higher than in the pooling equilibrium.

In the model with fertility, the results are slightly different from the simple model. Expected expenditures on education are the same in the two equilibria. However, similar to what we mentioned above, fertility for the pooling and separating choices in each own regime is higher than otherwise. Considering the higher probability, the fertility is higher in the separating equilibrium than in the pooling equilibrium, leading to higher expected expenditures on education for all children in the separating equilibrium. Finally, fertility has two conflicting forces in probability and parental human capital. Which force is greater is ambiguous, but empirically, negative effects of parental human capital on fertility likely dominate.

Though this paper contributes to exploring a parent's educational and fertility choice for the heterogeneity of children's ability, it is pointed out that there are unrealistic problems with the model. Unlike Acemoglu's (1999) model, if a parent is altruistic, she is unlikely to abandon her child, even when she observes the born child's low ability. In practice, she will not give up educating her low-ability child. In other words, it may seem difficult to observe a parent's separating choice in a real world, and thus this unrealistic problem would undercut the conclusions in this paper. Therefore, further research is required. Ideally, this research will study parents in the real world making choices about fertility and the education of children with heterogeneous ability.

Appendix

Proof of Proposition 1

We can prove by comparing the maximal values of the parent's problem for each state. To do this, we compare the maximal values of her pooling choices ($d^H = d^L = d$) in Equation (7). For instance, consider the maximal value $V^p \equiv V(e^*_{t+1}, d^H = 1, d^L = 1) = w_t h_t(1-\theta) + \frac{1-\alpha}{\alpha}\left[\alpha\beta h_t^{\delta}\{\lambda\kappa^{\gamma} +\right.$ $1{-}\lambda 1 1{-}\alpha$. It is straightforward to see that the value, Vp, for this choice ($d{=}1$) is greater than for any other pooling choices ($d < 1$) because $\alpha < 1$. Accordingly, the pooling choice $(d^H, d^L) = (1,1)$ is the equilibrium in the pooling regime. On the other hand, consider a separating choice of $V^s \equiv V(e^*_{t+1}, d^H = 1, d^L = 0) = w_t h_t(1-\theta) + \frac{1-\alpha}{\alpha}\lambda\left[\alpha\beta\, h_t^{\delta}\kappa^{\gamma}\right]^{\frac{1}{1-\alpha}}$. Note that the condition $\alpha < 1$ also implies $V^s > V(e^*_{t+1}, d^H < 1, d^L = 0)$.

For V^s to be greater than V^p, the condition, $\frac{\lambda\kappa^{\gamma}+(1-\lambda)}{\lambda^{1-\alpha}\kappa^{\gamma}} < 1$, should be satisfied. Simplifying this inequality would give us the following condition: $\kappa > \left[\frac{1-\lambda}{\lambda^{1-\alpha}-\lambda}\right]^{\frac{1}{\gamma}}$. If this condition holds, we can obtain $V^s > V(e^*_{t+1}, d^H \leq 1, d^L < 1)$. The separating choice $(d^H, d^L) = (1{,}0)$ is, in turn, the unique equilibrium. In contrast, if $\kappa < \left[\frac{1-\lambda}{\lambda^{1-\alpha}-\lambda}\right]^{\frac{1}{\gamma}}$, then we can get $V^p > V(e^*_{t+1}, d^H \leq 1, d^L < 1)$, concluding that the pooling choice $(d^H, d^L) = (1{,}1)$ is the unique equilibrium. Q.E.D.

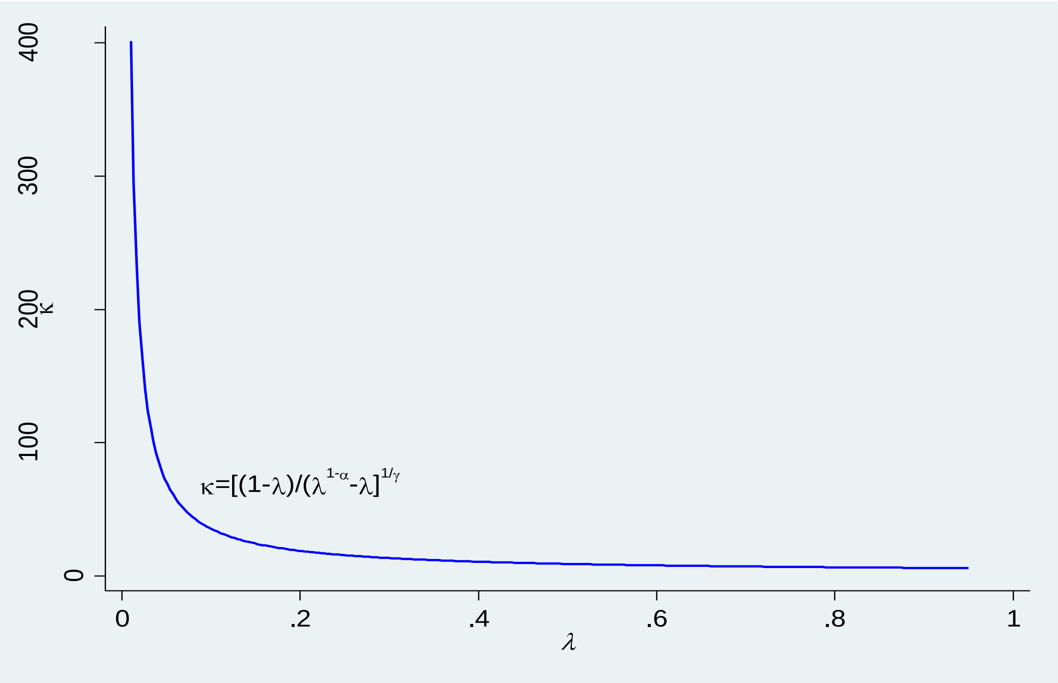

Figure 1: Two equilibria and the critical condition. $\alpha = 0.5$ and $\gamma = 0.4$.

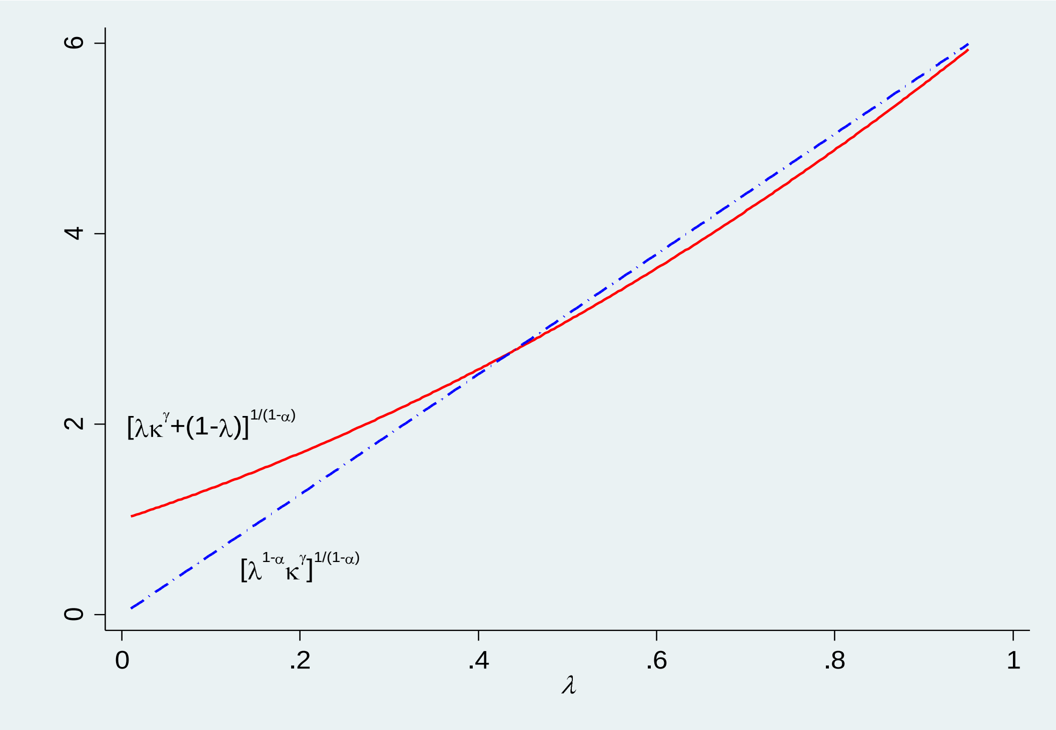

Figure 2: Standardized expected educational expenditure. $\alpha\beta h_t^{\delta} = 1$, $\alpha = 0.5$, $\gamma = 0.4$, and $\kappa = 10$.

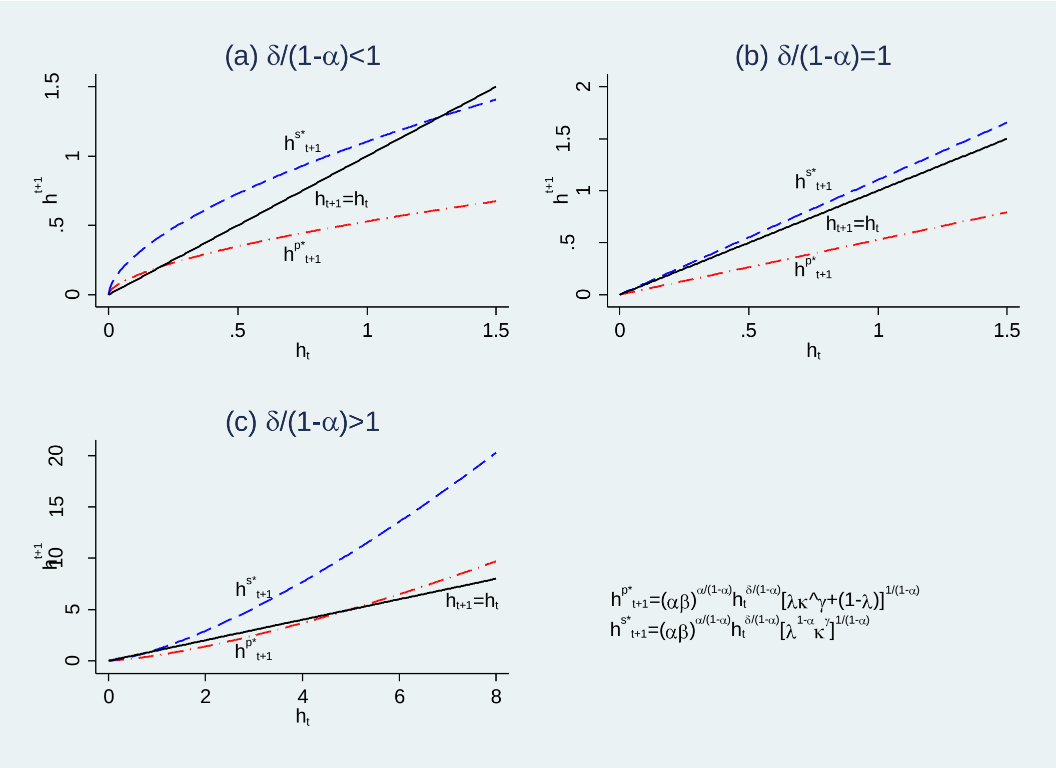

Figure 3: Dynamics of human capital. $\alpha = 0.5$, $\beta = 0.5$, $\gamma = 0.4$, $\kappa = 10$, $\lambda_1 = 0.3$, and $\lambda_2 = 0.7$.

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